



Early Journal Content on JSTOR, Free to Anyone in the World

This article is one of nearly 500,000 scholarly works digitized and made freely available to everyone in the world by JSTOR.

Known as the Early Journal Content, this set of works include research articles, news, letters, and other writings published in more than 200 of the oldest leading academic journals. The works date from the mid-seventeenth to the early twentieth centuries.

We encourage people to read and share the Early Journal Content openly and to tell others that this resource exists. People may post this content online or redistribute in any way for non-commercial purposes.

Read more about Early Journal Content at <http://about.jstor.org/participate-jstor/individuals/early-journal-content>.

JSTOR is a digital library of academic journals, books, and primary source objects. JSTOR helps people discover, use, and build upon a wide range of content through a powerful research and teaching platform, and preserves this content for future generations. JSTOR is part of ITHAKA, a not-for-profit organization that also includes Ithaka S+R and Portico. For more information about JSTOR, please contact support@jstor.org.

$$\therefore \angle BCD = \tan^{-1} \frac{-bc}{c^2 - a^2 - ab},$$

which will be a maximum when $c^2 - a^2 - ab = 0$, or $c^2 = a(a + b)$.

Q. E. D.

95. Proposed by WILLIAM HOOVER, A. M., Ph. D., Professor of Mathematics and Astronomy in Ohio State University, Athens, Ohio.

At each point of a parabola is described the rectangular hyperbola of a four-pointic contact; prove that the locus of the center of the hyperbola is an equal parabola.

Solution by the PROPOSER.

The curve having four-pointic contact with the parabola $y^2 = 4ax$ (1)

$$\text{is } y^2 - 4ax - \lambda(yy' - 2ax - 2ax')^2 = 0 \text{ (2),}$$

$$\text{or, } -4\lambda a^2 x^2 + 4\lambda a y' xy + (1 - \lambda y'^2) y^2 - (4a + 8a^2 \lambda x') x + 4a \lambda x' y' y - 4a^2 \lambda x'^2 = 0 \text{ . . (3).}$$

If this be an equilateral hyperbola,

$$4\lambda a^2 = 1 - \lambda y'^2, \text{ or } \lambda = 1 \div (y'^2 + 4a^2) \text{ (4).}$$

Substituting this in (2) and reducing,

$$ax^2 - y'xy - ay^2 + (4a^2 + y'^2 + 2ax')x - x'y'y + ax'^2 = 0 \text{ (5).}$$

The center of this is given by

$$x = -(y'^2 + 8a^2)/4a \text{ (6), } y = y' \text{ (7). } (x', y') \text{ being on (1), } y'^2 = 4ax' \text{ (8).}$$

Eliminating x', y' from (6), (7), and (8), we have the required locus,

$$y^2 = -4a(x + 2a) \text{ (9).}$$

96. Proposed by W. F. BRADBURY, A. M., Head Master, Cambridge Latin School, Cambridge, Mass.

Isosceles triangles are constructed externally on the three sides of a triangle as bases, with the angles at the bases each 30° . The triangle formed by joining the remote vertices (the 120° vertices) of these isosceles triangles is equilateral. [Geometric—not Trigonometric—solution.]

Solution by J. K. ELLWOOD, A. M., Principal of Colfax School, Pittsburg, Pa.

The vertices O , O_1 , O_2 , of the isosceles triangles are the centers of equilateral triangles described on the sides of ABC . Circumferences passed about these triangles intersect in a point, P .

Let P be the intersection of the two circles, AFC and CEB . Join AP , BP , and CP . Since $APCF$ is inscribed, $\angle F + \angle APC = 180^\circ$. But $\angle F = 60^\circ$.

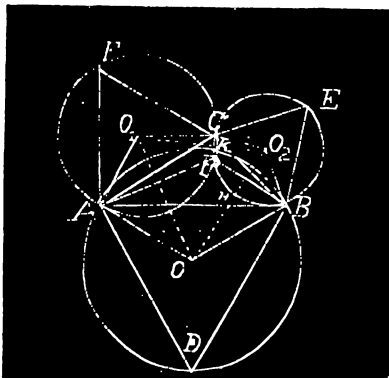
$$\therefore \angle APC = 120^\circ.$$

Similarly, $\angle CPB = 120^\circ$.

$$\therefore \angle APB = 120^\circ; \text{ and } \angle APB + \angle D = 180^\circ.$$

$\therefore APBD$ is inscribed, and P is in the circumference of ADB . Q. E. D.

Lines that join the centers of intersecting circles bisect the common



chords and the intercepted arcs. $\therefore$ arc $HP = \frac{1}{2}$ arc AP , and arc $PK = \frac{1}{2}$ arc PC .

$\therefore$ arc $HPK = \frac{1}{2}$ arc APC . But arc APC measures $\angle F = 60^\circ$ at circumference; therefore HK , its half, measures an equal angle at the center, and $\angle O_2 = \angle F = 60^\circ$. Similarly, $\angle O_1$ may be shown $= \angle E$, and $\angle O = \angle D$. Since equiangular triangles are equilateral, OO_1O_2 is equilateral. Q. E. D.

Solved in a similar manner by G. I. HOPKINS, F. R. HONEY, J. SCHEFFER, C. A. JONES, and G. B. M. ZERR.

CALCULUS.

73. Proposed by MOSES COBB STEVENS, A. M., Professor of Mathematics, Purdue University, Lafayette, Ind.

Solve $\int_0^{\frac{1}{2}\pi} \log(1 + \tan x) dx$.

I. Solution by G. B. M. ZERR, A. M., Ph. D., Professor of Science, Chester High School, Chester, Pa.

$$\begin{aligned} \int_0^{\frac{1}{2}\pi} \log(1 + \tan x) dx &= \int_0^{\frac{1}{2}\pi} \log[1 + \tan(\tfrac{1}{2}\pi - x)] dx = \int_0^{\frac{1}{2}\pi} \log\left(1 + \frac{1 - \tan x}{1 + \tan x}\right) dx \\ &= \int_0^{\frac{1}{2}\pi} \log\left(\frac{2}{1 + \tan x}\right) dx = \int_0^{\frac{1}{2}\pi} \log 2 \cdot dx - \int_0^{\frac{1}{2}\pi} \log(1 + \tan x) dx. \\ \therefore 2 \int_0^{\frac{1}{2}\pi} \log(1 + \tan x) dx &= \int_0^{\frac{1}{2}\pi} \log 2 \cdot dx = \tfrac{1}{2}\pi \log 2. \end{aligned}$$

$$\therefore \int_0^{\frac{1}{2}\pi} \log(1 + \tan x) dx = \tfrac{1}{4}\pi \log 2.$$

(See *Todhunter's Integral Calculus*, Art. 51, page 66.)

II. Solution by T. A. CLARK, of the Senior Class, Purdue University, Lafayette, Ind.

$$\begin{aligned} \int_0^{\frac{1}{2}\pi} \log(1 + \tan x) dx &= \int_0^{\frac{1}{2}\pi} \log(\tan \tfrac{1}{2}\pi + \tan x) dx = \int_0^{\frac{1}{2}\pi} \log \frac{\sin(\tfrac{1}{2}\pi + x)}{\cos \tfrac{1}{2}\pi \cos x} \\ &= \int_0^{\frac{1}{2}\pi} \log \sec \tfrac{1}{2}\pi dx + \int_0^{\frac{1}{2}\pi} \sin(\tfrac{1}{2}\pi + x) dx - \int_0^{\frac{1}{2}\pi} \log \cos x dx = \tfrac{1}{2}\pi \log 1/2. \end{aligned}$$

To prove $\int_0^{\frac{1}{2}\pi} \log \sin(\tfrac{1}{2}\pi + x) dx - \int_0^{\frac{1}{2}\pi} \log \cos x dx = 0$. Put $\tfrac{1}{2}\pi + x = \theta$, and

$$\tfrac{1}{2}\pi - x = \phi, \text{ then } \int_0^{\frac{1}{2}\pi} \log \sin(\tfrac{1}{2}\pi + x) dx = \int_{\frac{1}{2}\pi}^{\pi} \log \sin \theta d\theta, \text{ and } \int_0^{\frac{1}{2}\pi} \log \cos x dx$$